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# Tuning Maps for Setpoint Changes and Load Disturbance Upsets in a Three Capacity Process Under Multivariable Control

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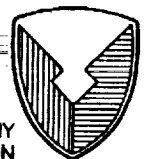
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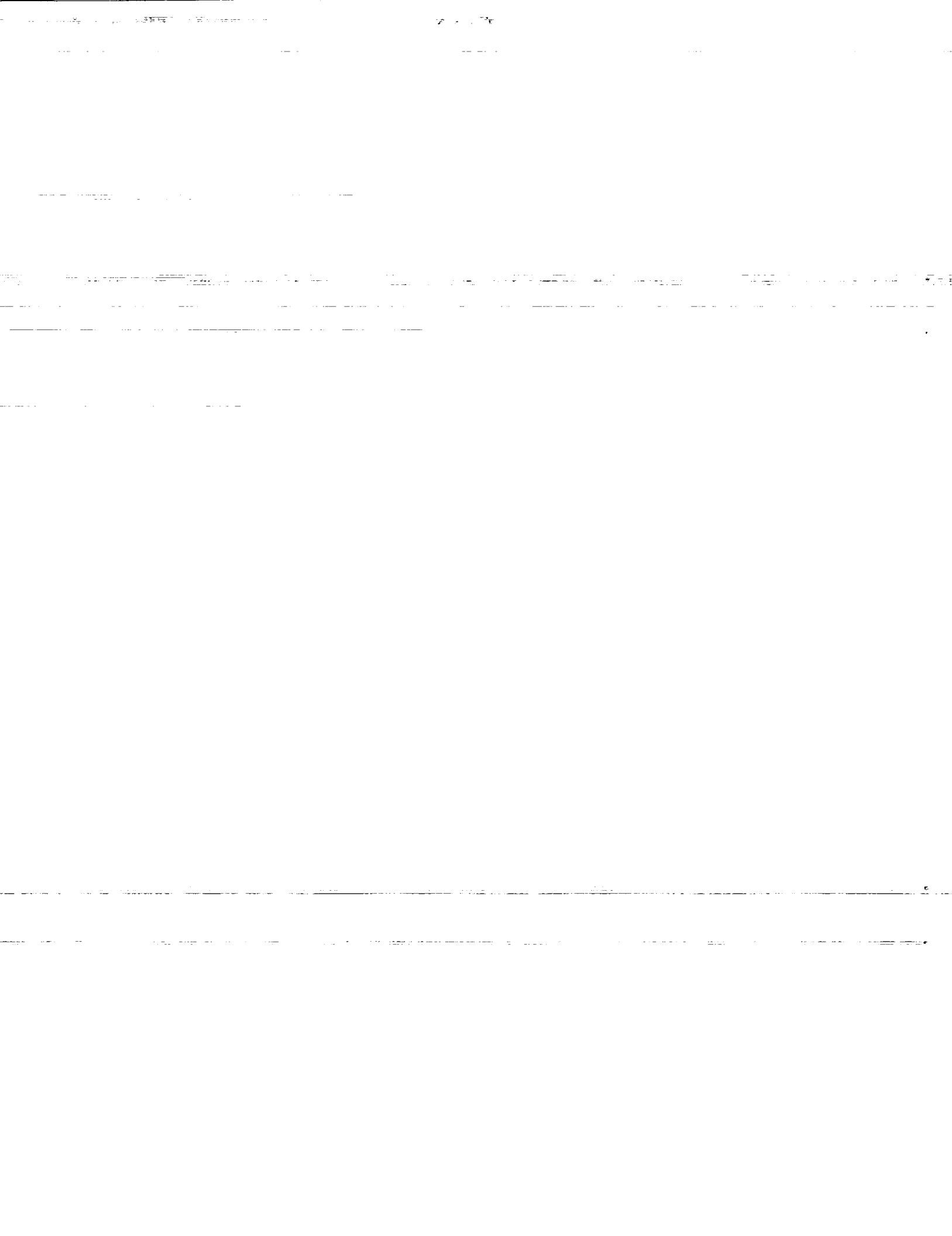


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TUNING MAPS FOR SETPOINT CHANGES AND LOAD DISTURBANCE UPSETS  
IN A THREE CAPACITY PROCESS UNDER MULTIVARIABLE CONTROL

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SUMMARY

Tuning maps are an aid in the controller tuning process because they provide a convenient way for the plant operator to determine the consequences of adjusting different controller parameters. In this application the maps provide a graphical representation of the effect of varying the gains in the state feedback matrix on startup and load disturbance transients for a three capacity process. Nominally, the three tank system, represented in diagonal form, has a Proportional-Integral control on each loop. Cross-coupling is then introduced between the loops by using nonzero off-diagonal proportional parameters. Changes in transient behavior due to setpoint and load changes are examined by varying the gains of the cross-coupling terms.

INTRODUCTION

Tuning of control systems to achieve desired transient response characteristics to setpoint and load changes has long been an important issue. In 1942, Ziegler and Nichols proposed the first structured approach to the tuning of Proportional-Integral-Derivative (PID) controllers to appear in the literature (ref. 1). This technique presented a simple way to achieve one-quarter damping in response to load disturbance upsets in industrial-type single-input-single-output processes. Since then, many forms of the algorithm have appeared with the emphasis on different types of responses (refs. 2 to 6). As modern control techniques have become more popular in practice, some tuning methods for multivariable systems have begun to appear in the literature (refs. 7 to 9).

The original, pre-Ziegler-Nichols tuning method, sometimes known as tweaking or trimming, is still very much in use in industry. It consists of an operator observing a transient, adjusting the controller gains based on experience, and rerunning the transient. This method is often limited in practice by the infeasibility of running exploratory step responses on-line and by the length of the dominant time constant of the plant which may be hours for some processes. During normal plant operation, an operator is generally more concerned with load disturbance rejection than with response to setpoint changes. However, since it is often impractical to generate load

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upsets for testing purposes, tuning of a controller to compensate for loads may be difficult. Thus, operator dissatisfaction with closed loop performance is common and results in about half of all industrial control loops being run on manual (in open-loop mode) (ref. 10).

Operator aids known as tuning maps were presented in 1962 (refs. 11 and 12). These maps consisted of transient responses of a specified shape (one-quarter damping, for instance) plotted in the plane comprised of the normalized integral and derivative gains. The loop was closed around a multiple lag plus deadtime process with a PID controller. The integral and derivative terms were fixed and the proportional value was varied until the step response had the desired characteristics. The transient was displayed at the appropriate point in the normalized Integral-Derivative plane and the proportional value which produced the response was shown alongside it. The map allowed the operator of a process with the designated transfer function to choose the PID values directly from the map of the desired response curve shape.

The advantage of the tuning map over the Ziegler-Nichols-type tuning algorithms is that it gives a large choice of PID parameters rather than a single set for the desired response shape. Since the Ziegler-Nichols algorithm seeks to achieve only one feature of the response (damping) as its objective, other parameters such as period of oscillation, percent overshoot, etc., are not considered. Operators who must meet several of these constraints simultaneously need the flexibility to choose initial controller values which will come close to attaining their goals at once.

An extension of the tuning map idea which offers even more flexibility is the three-dimensional tuning map. This map again contains transient plots at specific points in the normalized Integral-Derivative plane but with many proportional values used for each Integral-Derivative pair. Thus, the map gives the additional information of the direction of adjustment for the tuning parameters to attain the desired response. This technique allows an overall response curve to be chosen rather than simply a single feature (ref. 13).

All previous work has been applied to single-loop processes using classical control methodologies. Our work involves using tuning maps for a state feedback controller to demonstrate the effect of introducing interaction between multiple loops of a formerly decoupled process. Generally it is desirable to have noninteracting loops for ease of tuning, but, if the interaction provides the potential for improved system response and the method for tuning the controller is provided, the addition of interaction can be beneficial. The sample process used consisted of three noninteracting cascaded capacities. This system was selected arbitrarily as a representative industrial process. The scope of the work was limited to having at most one cross-coupling term in each multivariable controller examined. It was felt that exploring more complicated interactions would obscure the purpose of the maps by creating an unwieldy number of variables which would overwhelm rather than aid the operator (ref. 14).

The remainder of the paper consists of three parts: a description of the process and the artificial interactions imposed on it; a summary and analysis of the tuning maps obtained for the process; and conclusions about the utility of the maps.

### THREE CAPACITY PROCESS

The three tank system shown in figure 1 is an example of the noninteracting three capacity process of interest. We will assume that each tank is tall enough with enough liquid that transient responses will not cause any tank to empty or overflow, and that the valve determining the flow out of the tank will never fully open or fully close. These restrictions will assure linear behavior. We will also assume that the level in each tank is regulated about its own setpoint. Finally, we are concerned only with the level of the bottom tank. The levels of the upper two are measured as intermediate variables for control purposes only, they are not system outputs.

The differential equation representing the liquid level,  $h$ , inside each tank is a function of the inflow,  $q_{in}$ , and outflow,  $q_{out}$ , namely,

$$A\dot{h}(t) = q_{in}(t) - q_{out}(t)$$

where  $A$  is the cross-sectional area of the tank which is assumed to be unity.

We assume that the outflow is regulated by a valve which can convert a flow command to an appropriate opening in negligible time. Thus the transfer function of the valve from flow request to output is

$$G_v(s) = 1.0 \quad (1)$$

A Proportional-Integral controller of the form

$$G_c(s) = k_p + \frac{k_I}{s} = \frac{k_p s + k_I}{s} \quad (2)$$

is used to govern the level in the tank. The closed loop transfer function for any one tank is

$$h(s) = \frac{r}{s} + \frac{s}{s^2 + k_p s + k_I} \Delta r \quad (3)$$

where  $r$  represents the new setpoint for the liquid level in the tank which we assume is a constant, and  $\Delta r$  is the amount of the step change in setpoint (see the appendix for the derivation). Figure 2 depicts a block diagram of one of the tanks. The transfer function of the error in the level (actual minus desired) of any one tank is simply

$$e(s) = \frac{s}{s^2 + k_p s + k_I} \Delta r$$

assuming the disturbance input,  $q_{in}(t)$ , is unchanged. In the case of a setpoint change at the lowest tank only, the transfer function for the level in that tank is given by equation (3). As long as the capacities are

noninteracting, the Proportional-Integral controller for the lowest tank determines the transient response for the level as it is adjusted due to the setpoint change.

For load disturbance upsets,  $\Delta q_{in}$ , the closed loop transfer function for any individual tank is

$$h(s) = \frac{r}{s} + \frac{s}{s^2 + k_p s + k_I} \Delta q_{in}(s) \quad (4)$$

where  $r$  represents the setpoint for the liquid level in the tank and we assume that it is a constant. The transfer function of the error in the level (actual minus desired) of any one tank is simply

$$e(s) = \frac{s}{s^2 + k_p s + k_I} \Delta q_{in}(s) \quad (5)$$

From figure 2 it can be seen, using equations (1), (2), and (5), that

$$q_{out}(s) = \frac{k_p s + k_I}{s^2 + k_p s + k_I} \Delta q_{in}(s) \quad (6)$$

Therefore, in the case of a flow change,  $\Delta v$ , into the highest tank, the transfer function for the level in the lowest tank is

$$h(s) = \frac{r}{s} + \frac{s}{s^2 + k_p s + k_I} \frac{k_p s + k_I}{s^2 + k_p s + k_I} \frac{k_p s + k_I}{s^2 + k_p s + k_I} \Delta v(s)$$

which is a combination of equations (4) and (6) since the outflow of one tank is the flow into the next, causing the error to propagate through the three noninteracting tanks.

Using the modern control technique of state feedback to, in effect, couple the capacities, it might be possible to improve the transient response to setpoint or load changes by restricting or increasing the flow from the higher tanks to the bottom one.

A possible state space realization for the three tank process is

$$\begin{aligned}
 \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{bmatrix} &= Ah + B_q q_{out} + Fv = A(e + r) + B_q q_{out} + Fv \\
 &= Ae + Ar + B_q q_{out} + Fv = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} q_{out1} \\ q_{out2} \\ q_{out3} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v \\
 &= Ae + B_q q_{out} + Fv = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} q_{out1} \\ q_{out2} \\ q_{out3} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v \\
 h_3 &= Ch = [0 \ 0 \ 1] \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}
 \end{aligned}$$

where  $v$  is the unrestricted flow into the top tank.

In order to include the integral terms of the controllers, the state vector can be augmented by the integrator state variables,  $x_1$ ,  $x_2$ , and  $x_3$ . These additional variables provide the integral of the error in the level of each tank. The sixth order equation is

$$\begin{aligned}
 \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{out1} \\ q_{out2} \\ q_{out3} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} v \\
 h_3 &= [0 \ 0 \ 1 \ 0 \ 0 \ 0] \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}
 \end{aligned} \tag{7}$$

If we introduce the state feedback matrix,  $K = [K_p \ K_I]$ , which incorporates the proportional and integral gains, we can obtain a closed loop system of the form

$$\begin{bmatrix} \dot{h} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A & 0 \\ I & 0 \end{bmatrix} \begin{bmatrix} e \\ x \end{bmatrix} + \begin{bmatrix} B_q \\ 0 \end{bmatrix} \begin{bmatrix} K_p & K_I \end{bmatrix} \begin{bmatrix} e \\ x \end{bmatrix} + \begin{bmatrix} F \\ 0 \end{bmatrix} V = \begin{bmatrix} A + B_q K_p & B_q K_I \\ I & 0 \end{bmatrix} \begin{bmatrix} e \\ x \end{bmatrix} + \begin{bmatrix} F \\ 0 \end{bmatrix} V$$

by setting the vector  $q_{out}$  from equation (7) to

$$\begin{bmatrix} q_{out_1} \\ q_{out_2} \\ q_{out_3} \end{bmatrix} = [K_p \ K_I] \begin{bmatrix} e \\ x \end{bmatrix} = \begin{bmatrix} k_{p_{11}} & 0 & 0 & k_{I_{11}} & 0 & 0 \\ 0 & k_{p_{22}} & 0 & 0 & k_{I_{22}} & 0 \\ 0 & 0 & k_{p_{33}} & 0 & 0 & k_{I_{33}} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (8)$$

This gives us a realization of the three noninteracting capacities pictured in figure 1. The fact that the portions of the  $K$  matrix corresponding to the proportional and integral parts of the controller are diagonal keeps the tanks decoupled.

Introducing interaction between the tanks in order to improve the startup or load disturbance rejection characteristics involves simply adding nonzero off-diagonal terms to the  $K$  matrix. In this work we are concerned only with the effect of proportional coupling on the system. The state space formulation of the interacting system is

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_{p_{11}} & k_{p_{12}} & k_{p_{13}} & k_{I_{11}} & 0 & 0 \\ k_{p_{21}} & k_{p_{22}} & k_{p_{23}} & 0 & k_{I_{22}} & 0 \\ k_{p_{31}} & k_{p_{32}} & k_{p_{33}} & 0 & 0 & k_{I_{33}} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} V$$



$$\begin{aligned}
 \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} -k_{P11} & -k_{P12} & -k_{P13} & -k_{I11} & 0 & 0 \\ k_{P11} & -k_{P21} & k_{P12} & -k_{P22} & k_{P13} & -k_{P23} & k_{I11} & -k_{I22} & 0 \\ k_{P21} & -k_{P31} & k_{P22} & -k_{P32} & k_{P23} & -k_{P33} & 0 & k_{I22} & -k_{I33} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} v \\
 h_3 &= [0 \ 0 \ 1 \ 0 \ 0 \ 0] \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}
 \end{aligned} \tag{9}$$

The tuning technique we will present may be understood symbolically from figure 3. Each knob on the front of the box in figure 3 represents a proportional element of the left half of the  $K$  matrix. The tube on top provides the disturbance input to the uppermost tank. The setpoints for the tanks enter on the left. The process starts out in steady state. The output due to a step change in the level command or the disturbance input is a transient response which is plotted on a strip chart recorder to the right. Many transients are obtained, each with the knobs adjusted differently. The plots are collected and presented in a way which makes it easy to see the effect of each tuning change on the output.

Figures 4 to 6 show the tuning maps generated for step setpoint changes from the system in equation (9). Figure 7 depicts the tuning map created for the system in (9) due to a step load disturbance. The three-by-three grids of plots are arranged so that each set of graphs in each figure corresponds to the adjustment of the corresponding tuning knob from figure 3. In each of these figures, the nominal or noninteracting system (the combination of equations (7) and (8) with  $k_{I11}=1$  and  $k_{P11}=2$ ) is used. One by one, each proportional element of the  $K$  matrix is varied through a range of values and then reset to its nominal value. Each set of graphs depicts step responses to setpoint or load changes. Within each graph, the corresponding proportional gain is increased from upper left to lower right. In figure 4 the setpoint of the bottom tank only is changed; in figure 5 the desired levels of the bottom two tanks are changed simultaneously; and in figure 6 all three requests are altered concurrently.

## RESULTS

The tuning maps in figures 4 to 6 each represent different situations. The case presented in figure 4 concerns a setpoint change in the bottom tank only. Consequently, since the process is originally in steady state, feeding the error from either of the upper two tanks to any other loop has no effect on the transient response. This is because no error has been introduced in the levels of the upper two tanks. The result is that the tuning maps vary with the element of  $K$  in the right column only. Likewise, in figure 5 where the command is changed in the lower two tanks, feeding the error in the level of the uppermost tank to either of the lower two has no effect since the error is zero. This results in the left column of the map being unaffected by variations in  $K$ . Figure 6 shows the case where all setpoints are changed making any change in  $K$  have an effect on the step response transient.

The tuning map in figure 7 represents the case where a step change is introduced in the flow into the top tank. This disturbance upsets the equilibrium in the tanks, and the valves controlling outflow must be adjusted to return the tanks to their desired levels. The plots depict the impact of altering the interaction between the tanks on the level in the bottom tank. Since the error is injected at the beginning of the flow path and its effect is propagated through to the end, any change in the interaction between upsets causes the ensuing transient responses to differ.

One caveat concerns the settings of parameters which apparently have no effect on the transients in figures 4 and 5. The fact that no changes occur in the step response due to varying the  $K$  value does not mean that the gain selection is arbitrary; only when the error is exactly zero will this situation arise. Uninformed choice of the gains could result in an unstable system which will manifest itself only due to noise or disturbance upsets. In this case, any nonzero error will cause a runaway in the unstable loop, creating a serious upset in the output. Since the output of this three capacity process is ultimately limited by flow into the top tank, the desired output steady state level might be achieved but the transient response will be unpredictable, defeating the purpose of the tuning map.

Also, the responses from the figures due to varying particular  $K$  elements away from the nominal form are in general not additive. That is, altering two or more  $K$  values at once in order to achieve, in some sense, a combination of two responses from the maps will not work and may, in fact, give an unstable result. It is entirely possible that simultaneously altering several gains from their nominal settings will give greatly improved results. However, the tuning maps presented here will not provide the information on how to accomplish this.

## CONCLUSIONS

Tuning maps for a three capacity process under multivariable control were developed as a guide for choosing gains of a state feedback controller. The maps provide a concise graphical representation of the effect of altering a single proportional gain on startup and load disturbance transients. The use of the maps is straightforward and easily understood.

The difficulties discussed at the end of the results section are an indication of the complexity encountered when introducing interaction to even a simple system. The maps are intended to be used as a guide during the tuning process, not as a design aid. As such, they demonstrate the effect of simple parameter changes on step response transients and make no attempt to address the result of changing several gains at once.

The determination of what constitutes the best transient response to a setpoint or load change basically depends upon the operator's preferences. There may be some constraints tied to the process's output product quality, but otherwise the operator is generally free to choose the response shape. The use of multivariable tuning maps gives the operator an extra degree of freedom over standard single loop tuning methods and provides the potential for greatly improved control.

## APPENDIX

The derivation of the transfer function for the transient response of the level in a single tank to setpoint and load changes follows. Refer to figure 2 for a block diagram of a single tank.

We may begin with the differential equation for the level in the tank

$$A\dot{h}(t) = q_{in}(t) - q_{out}(t) \quad (A1)$$

and for simplicity we assume the cross sectional area,  $A$ , of the tank is unity.

If we assume that the system is in steady state before the setpoint and load changes, we have the relation that

$$\dot{h}(t) = 0$$

which, combined with equation (A1), gives us

$$q_{in}(t) = q_{out}(t) \quad \text{or} \quad q_{in}(t_0-) = q_{out}(t_0-) \quad (A2)$$

where  $t_0-$  is the time just before the setpoint and load changes.

Referring to figure 2 we see that

$$q_{out}(s) = \frac{k_p s + k_I}{s} (h(s) - r(s))$$

which has the particular solution

$$\begin{aligned} s q_{out}(s) - q_{out}(t_0-) &= (k_p s + k_I) (h(s) - r(s)) \\ q_{out}(s) &= \frac{q_{out}(t_0-)}{s} + \frac{k_p s + k_I}{s} (h(s) - r(s)) \end{aligned} \quad (A3)$$

Immediately after the setpoint and load changes occur, the level in the tank,  $h(t_0+)$ , is the same as it was just before the setpoint change. Thus it is clear from figure 2 that

$$h(t_0-) = h(t_0+) = r + \Delta r \quad (A4)$$

where  $r$  is the new setpoint and  $\Delta r$  is the value of the step change in setpoint, equal to the error at time  $t_0+$ .

Finally, from time  $t_0+$  on, we have the fact that

$$r(s) = \frac{r}{s} \quad \text{and} \quad q_{in}(s) = \frac{q_{in}(t_0-)}{s} + \Delta q_{in}(s) \quad (A5)$$

where  $\Delta q_{in}$  is the change in load.

Transforming equation (A1) into the Laplace domain and substituting in equation (A3) gives

$$\begin{aligned} sh(s) - h(t_0^-) &= q_{in}(s) - q_{out}(s) \\ sh(s) &= h(t_0^-) + q_{in}(s) - \frac{q_{out}(t_0^-)}{s} - \frac{k_p s + k_I}{s} (h(s) - r(s)) \\ &= h(t_0^-) + q_{in}(s) - \frac{q_{out}(t_0^-)}{s} - \frac{k_p s + k_I}{s} h(s) + \frac{k_p s + k_I}{s} r(s) \end{aligned}$$

Substituting in equations (A4) and (A5) and rearranging gives

$$sh(s) + \frac{k_p s + k_I}{s} h(s) = \Delta r + r + \frac{q_{in}(t_0^-)}{s} + \Delta q_{in}(s) - \frac{q_{out}(t_0^-)}{s} + \frac{k_p s + k_I}{s} \frac{r}{s}$$

Using equation (A2) and combining terms gives

$$\begin{aligned} \frac{s^2 + k_p s + k_I}{s} h(s) &= \Delta r + \frac{q_{out}(t_0^-)}{s} + \Delta q_{in}(s) - \frac{q_{out}(t_0^-)}{s} + \frac{s^2 + k_p s + k_I}{s} \frac{r}{s} \\ h(s) &= \frac{s}{s^2 + k_p s + k_I} \Delta r + \frac{s}{s^2 + k_p s + k_I} \Delta q_{in}(s) + \frac{r}{s} \end{aligned}$$

## REFERENCES

1. Ziegler, J.G.; Nichols, N.B., "Optimum Settings for Automatic Controllers," Transactions of the A.S.M.E., Vol. 64, No. 11, (1942), pp. 759-768.
2. Pessen, D.W., "Optimum Three-Mode Controller Settings for Automatic Start-Up," Transactions of the A.S.M.E., Vol. 75, No. 7, (1953), pp. 843-849.
3. Pessen, D.W., "How to 'Tune-In' a Three-Mode Controller," Instrumentation, Vol. 7, No. 3, Second Quarter, 1954, pp. 29-32.
4. Hazebroek, P., Van Der Waerden, B.L., "The Optimal Adjustment of Regulators," Transactions of the A.S.M.E., Vol. 72, April, 1950, pp. 317-322.
5. Cohen, G.H.; Coon, G.A., "Theoretical Consideration of Retarded Control," Transactions of the A.S.M.E., Vol. 75, July, 1953, pp. 827-834.
6. Wolfe, W.A., "Controller Settings for Optimal Control," Transactions of the A.S.M.E., Vol. 73, May, 1951, pp. 413-418.
7. Niederlinski, A., "A Heuristic Approach to the Design of Linear Multivariable Interacting Control Systems," Automatica, Vol. 7, (1971), pp. 691-701.
8. Luyben, W.L., "Simple Method for Tuning SISO Controllers in Multivariable Systems," Industrial and Engineering Chemistry. Process Design and Development, Vol. 25, (1986), pp. 854-860.
9. Owens, D.H., "A Loop Tuning Condition for Multivariable Process Control Using Steady State Step Response Data," Control-Theory and Advanced Technology, Vol. 1, No. 4, December, 1985, pp. 267-274.
10. Andreiev, N., "A New Dimension: A Self-Tuning Controller That Continually Optimizes PID Constants," Control Engineering, August, 1981, pp. 84, 85.
11. Wills, D.M., "Tuning Maps for Three-Mode Controllers," Control Engineering, April, 1962, pp. 104-108.
12. Wills, D.M., "A Guide to Controller Tuning," Control Engineering, August, 1962, pp. 93-95.
13. Litt, J.S., "An Expert System to Perform On-Line Controller Tuning," 1990 American Control Conference, San Diego, CA, May 23-25, 1990.
14. Miller, G.A., "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Information Processing," Psychological Review, Vol. 63 (1956), pp. 81-97.

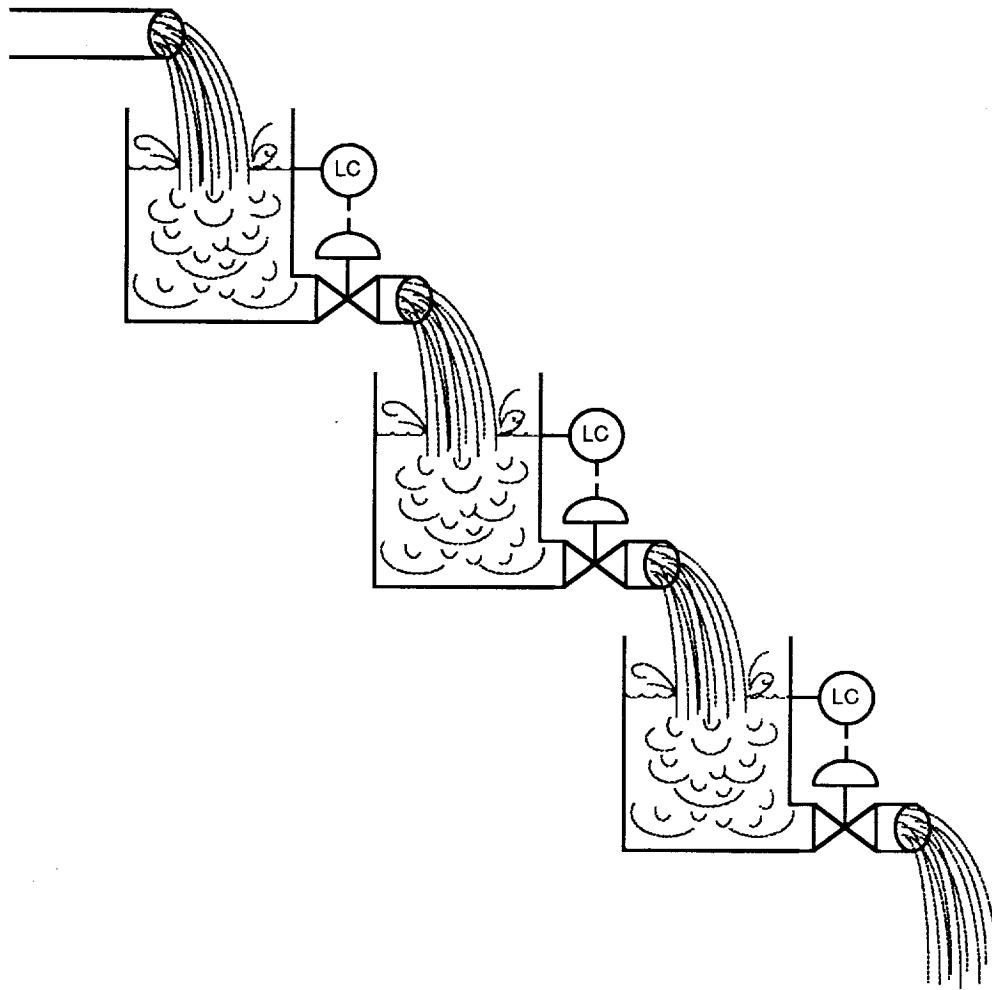


Figure 1.—A representation of a non-interacting three-capacity process.

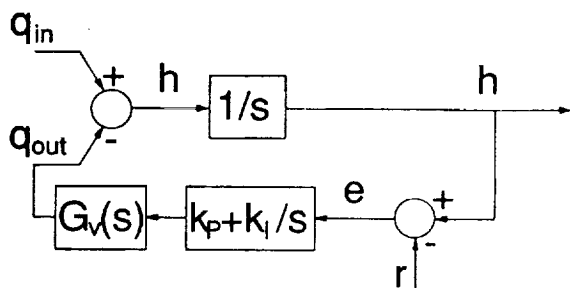


Figure 2.—Block diagram of a single tank under PI control.

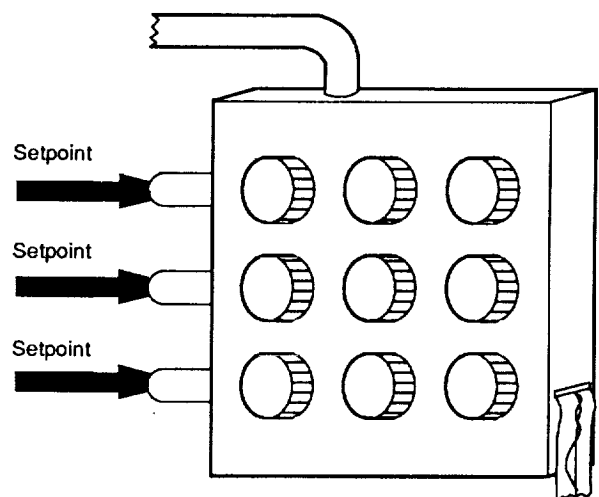


Figure 3.—Symbolic representation of multivariable controller tuning.

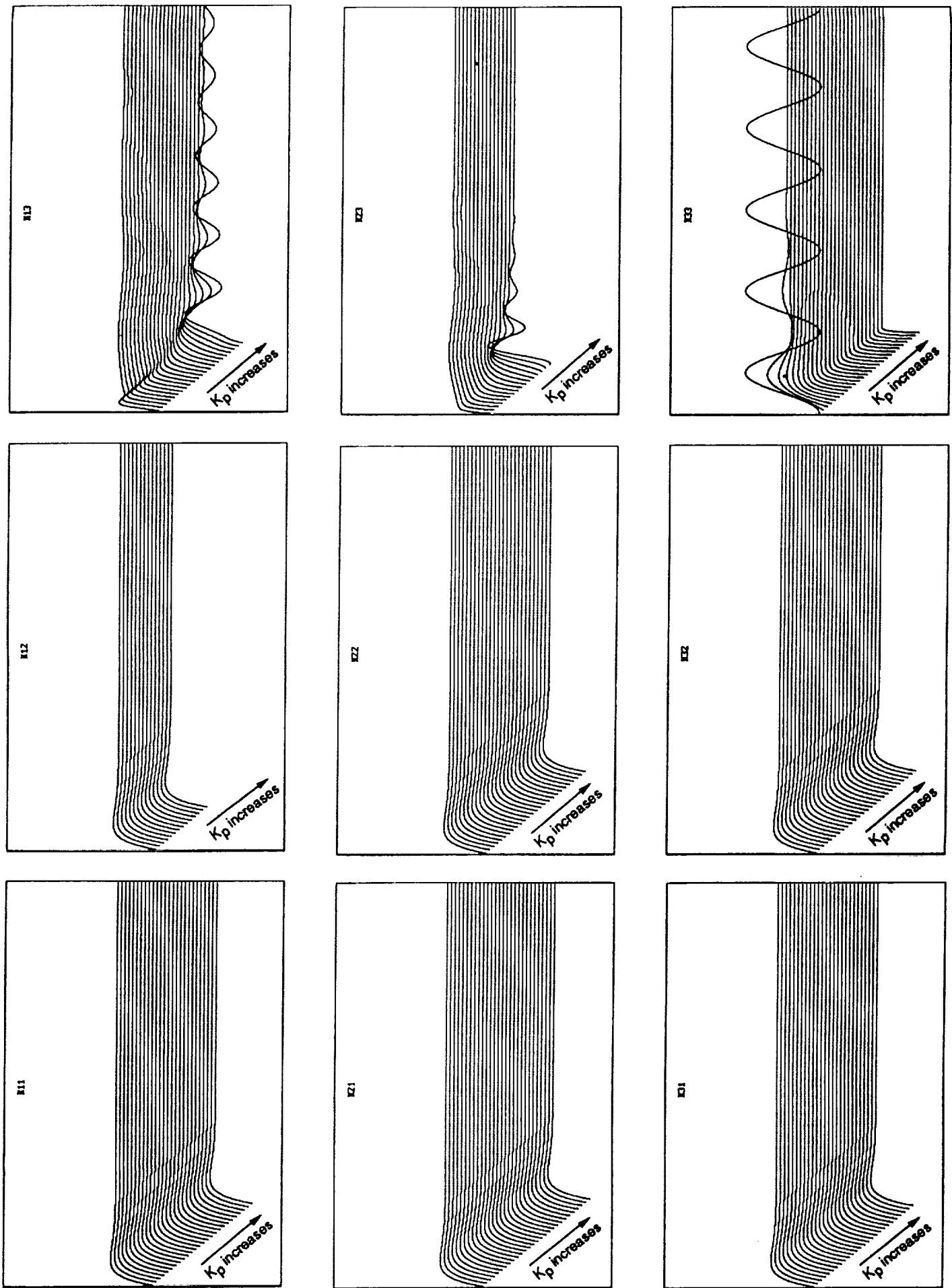


Figure 4.—Tuning map of the response of a three-capacity process under multivariable control to a set point change in the bottom tank.



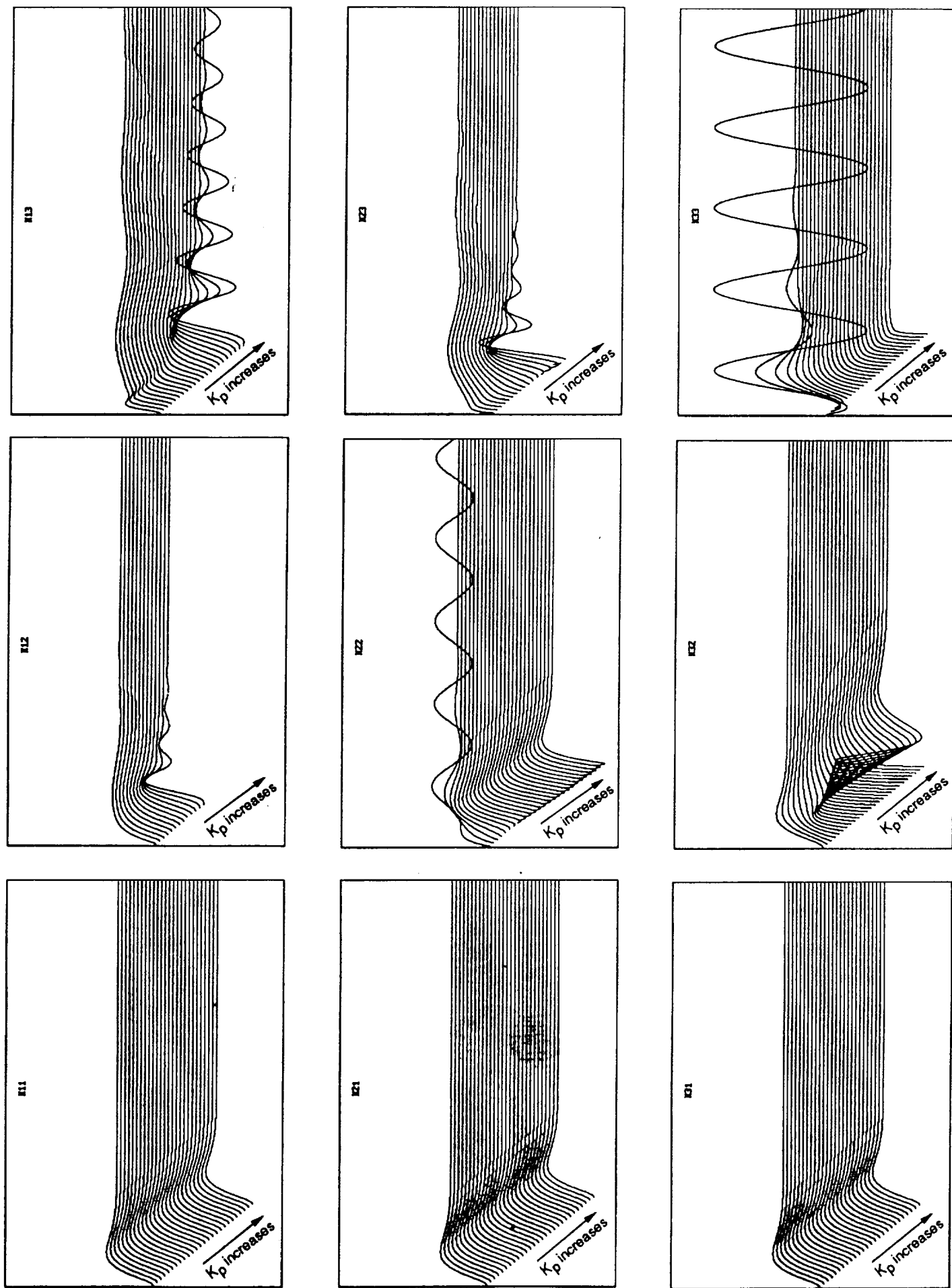


Figure 5.—Tuning map of the response of a three-capacity process under multivariable control to simultaneous set point changes in the bottom two tanks.

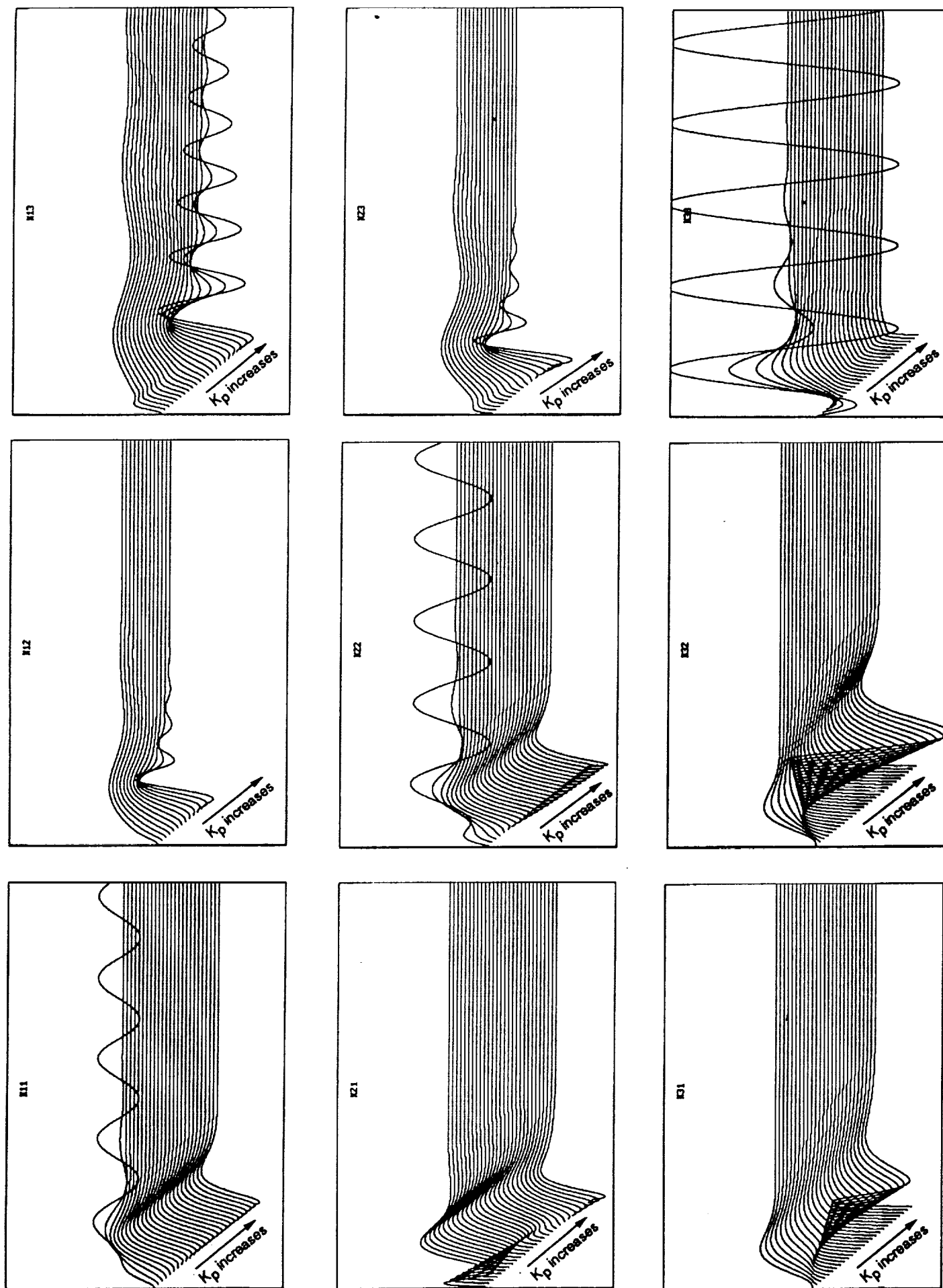


Figure 6.—Tuning map of the response of a three-capacity process under multivariable control to simultaneous set point changes in all three tanks.

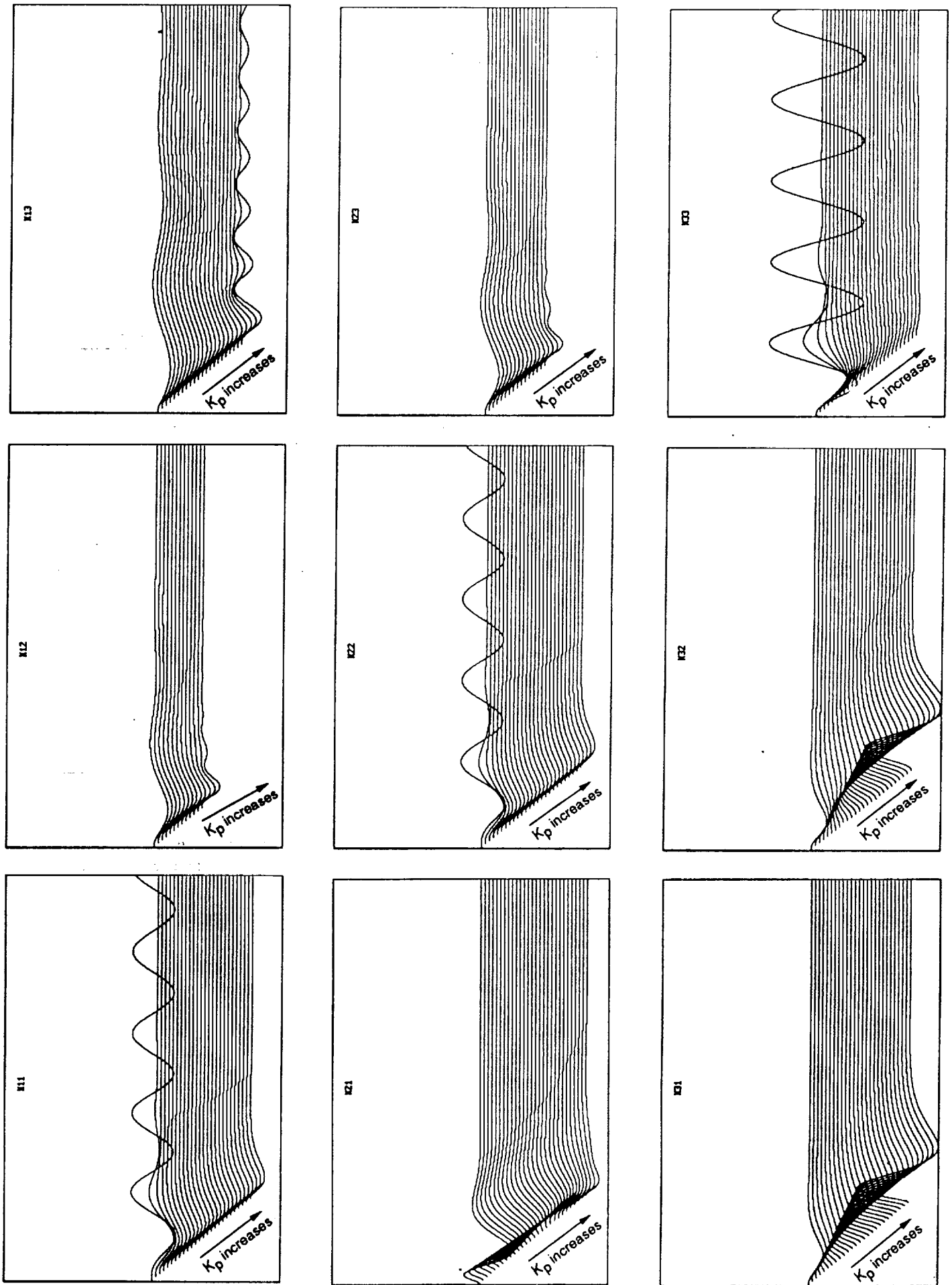


Figure 7.—Tuning map of the response of a three-capacity process under multivariable control to a load disturbance upset.



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16. Abstract Tuning maps are an aid in the controller tuning process because they provide a convenient way for the plant operator to determine the consequences of adjusting different controller parameters. In this application the maps provide a graphical representation of the effect of varying the gains in the state feedback matrix on startup and load disturbance transients for a three capacity process. Nominally, the three tank system, represented in diagonal form, has a Proportional-Integral control on each loop. Cross-coupling is then introduced between the loops by using nonzero off-diagonal proportional parameters. Changes in transient behavior due to setpoint and load changes are examined by varying the gains of the cross-coupling terms.					
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